III Semester M.Sc. Examination, January 2019
(CBCS – Y2K17 Scheme)
MATHEMATICS
M305T – Numerical Analysis – II

Time : 3 Hours Max. Marks : 70

Instructions : *a) Answer **any five full** questions.*
*b) Use 4- digits with rounding in **all** your numerical calculations.*

1. a) Make a discussion on the absolute stability of the classical Runge-Kutta explicit method of one slope. **4**

   b) Find by three and four iterations of the Picard method the solution y(0, 1) of the problem :

   $$\frac{dy}{dx} = x + y^2;\ y(0) = 1$$

   Compare the two solutions. **10**

2. a) Show that the Runge-Kutta methods are relatively stable. **7**

   b) Solve

   $$\frac{dy}{dx} = x + y^2;\ y(0) = 1$$

   by any Runge-Kutta method of two slopes. Choose $\Delta x = 0.05$ and obtain y(0.1). **7**

3. Derive Adam's predictor-corrector method of any order. Using the method solve

$$\frac{dy}{dx} = x + y^2;\ y(0) = 1$$

Obtain y(0. 6). Generate needed initial values by any single-step method. **14**

4. Solve the boundary value problem $x\frac{d^2y}{dx^2} + y = x^2; y(0) = 1, y(1) = 1.4$ by both shooting and cubic-spline methods. Choose an appropriate value of Δx. 14

5. a) Solve the Poisson equation

$$u_{xx} + u_{yy} = -10\,(x^2 + y^2 + 5),\ 0 < x, y < 1$$

Subject to

$$\left.\begin{array}{l} u(0, y) = 0 \\ u(1, y) = 0 \end{array}\right\},\ 0 \le y < 1$$

$$\left.\begin{array}{l} u(x, 0) = 0 \\ u(x, 1) = 1 \end{array}\right\},\ 0 \le x < 1$$

using the central difference approximation to both the space derivatives with $\Delta x = \Delta y = \frac{1}{3}$. Use any iterative method to find the solution (fourth iterate) of the resulting system of linear algebraic equations. 8

b) Replace the Poisson equation in question 5) a) by the Laplace equation $u_{xx} + u_{yy} = 0$, and solve it. Comparing the solutions of the two equations comment on the effect of the term $-10\,(x^2 + y^2 + 5)$ on the solution. 6

6. Consider

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2},\ 0 < x < 1,\ t > 0$$

Discretize the region of interest and approximate the equation using the following methods :

i) Schmidt,

ii) Laasonen,

iii) Dufort-Frankel and

iv) Crank-Nicolson

Classify them into explicit and implicit methods. Discuss the stability of anyone implicit method. 14

7. a) Write down the algorithm for implementing the alternate direction implicit method of solving two-dimensional heat or wave equation. **7**

b) Show that the explicit finite-difference method of solving the one-dimensional wave equation is conditionally stable. **7**

8. Solve

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$$

Subject to

$$\left.\begin{aligned} u(x, 0) &= x^2(1 - x^2) \\ \frac{\partial u}{\partial t}(x, 0) &= 0 \end{aligned}\right\}, 0 \le x \le 1$$

$$\left.\begin{aligned} u(0, t) &= 0 \\ u(1, t) &= 0 \end{aligned}\right\}, t \ge 0$$

by both explicit and implicit finite difference methods. Choose $\Delta x = \frac{1}{4}$ and $\Delta t = \frac{1}{64}$. Obtain the solution at the first time level. Comment on the two solutions.